PHYSICS

Time Allowed : 3 hours Full Marks : 100

All questions carry equal marks of 10 each.
Answer 10 (ten) questions
taking 5 (five) questions from each section.

SECTION - A

1. The Lagrangian of a particle in motion is given by
\[ L = \frac{1}{2} \left( M\dot{x}^2 + M\dot{y}^2 + M\dot{z}^2 \right) - (ax + by + cz) \]
Determine the corresponding Hamiltonian in terms of the position and momenta coordinates.

2. Prove that the path of a particle moving in a central force is a plane curve.

3. What are Four-Vectors? In what space do they operate? With respect to their transformations, what kind of vectors are four vectors? Give the expression for and explain the covariant form of momentum four-vector.

4. Explain Fresnel and Fraunhaufer diffraction and the difference between the two.

(Contd. 2)
5. Show that the consecutive terms of the multipole expansion of scalar potential are linearly independent.

6. For plane electromagnetic waves, prove that the flow of energy per unit area per unit time is given by the Poynting vector.

7. Show that electrical and magnetic fields are invariant under gauge transformations.

8. What are the dimensions of Planck’s constant? Given that \( \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{Js} \) and \( G = 6.674 \times 10^{-11} \text{m}^3\text{kg}^{-1}\text{s}^{-2} \), calculate the value of Planck Mass, Planck time and Planck energy.

9. Explain the concept of negative temperature using the example of nuclear spins in an external magnetic field. Show that negative temperatures are possible only for systems with finite number of energy states.
10. Prove that the following operators are Hermitian

(a) $X$  (b) $P_x$  and  (c) \[ \begin{pmatrix} 1 & 1 - i \\ 1 + i & 0 \end{pmatrix} \]

Also show that unitary operators preserve the inner product between the vectors that they act upon.

11. Prove that the energy levels of a linear harmonic oscillator are equally spaced.

12. Using Pauli Spin Matrices $\sigma_x$, $\sigma_y$ and $\sigma_z$ show that

\[ (\hat{\sigma} \hat{\rho})(\hat{\sigma} \hat{\rho}) = \hat{\mathbf{r}} \cdot \hat{\mathbf{p}} + i \hat{\mathbf{L}} \] where the symbols have their usual meaning.

13. Explain Franck-Condon principle with the help of an energy level diagram. What is the difference between Born-Openheimer approximation and Franck-Condon principle? In spectroscopy the Franck-Condon principle is used to explain what kind of transitions?

14. Explain the origin of Stokes and Anti-Stokes lines in the Raman spectra of molecules. Why are they equally spaced with respect to the Rayleigh line? What are the selection rules? Under what conditions will a vibrational transition be Raman active too?

(Contd. 4)
15. Explain the basic concept of Nuclear Magnetic Resonance (NMR). Explain, with a block diagram, the working of a NMR spectrometer.

16. Explain the concept of parity violation in weak processes. Explain an experiment that was performed to observe the same.

17. What are the conserved quantities in a weak process? Explain using the decay process. What is meant by electroweak unification? What are the various particles in the standard model?

18. Explain with the appropriate circuit diagrams the construction and working of the following CMOS based gates: (a) Inverter (b) NAND and (c) NOR.