MIZORAM PUBLIC SERVICE COMMISSION

TECHNICAL COMPETITIVE EXAMINATIONS FOR RECRUITMENT TO THE POST OF JUNIOR GRADE OF MIZORAM PLANNING, ECONOMICS & STATISTICAL SERVICE OCTOBER, 2015

MATHEMATICS
PAPER – II

Time Allowed : 3 hours

Full Marks : 100

Figures in the margin indicate full marks for the questions.

PART – A

Attempt all questions.

1. Choose the correct answer from the following: **(10×2=20)**

(i) The function $f(x,y) = y^{\frac{1}{2}}$ on $R: |x| \leq 1, |y| \leq 1$

(a) satisfies Lipschitz condition

(b) does not satisfy Lipschitz condition

(c) cannot test Lipschitz condition

(d) none of these

(ii) The value of $(1+i)^4\left(1+\frac{1}{i}\right)^4$ is

(a) 4

(b) 8

(c) 16

(d) 28

(iii) The complete integrals of the PDE $z = px + qy + p + q - pq$ is

(a) $z = ax + by + a - ab$

(b) $z = ax + by + b - ab$

(c) $z = ax + by - ab$

(d) $z = ax + by + a + b - ab$

(iv) The solution of the PDE

$p = (z + qy)^2$ by Jacobi's method is given by

(a) $u_1.u_2 = b$ (constant)

(b) $u_1.u_3 = b$ (constant)

(c) $u_2.u_3 = b$ (constant)

(d) none of these

(v) The integrating factor of the equation $y' - \frac{\tan y}{1+x} = (1+x)e^x$

(a) $\frac{1}{1+x}$

(b) $-\frac{1}{1+x}$

(c) $\frac{e^x}{1+x}$

(d) $e^x + 1$

(vi) The partial differential equation $\frac{\partial^2 u}{\partial y^2} - y\frac{\partial^2 u}{\partial x^2} = 0$ has

(a) Two families of real characteristics curves for $y < 0$.

(b) No real characteristics curves for $y > 0$.

(c) Vertical lines as family of characteristics curves for $y = 0$

(d) Branches of quadratic curve as characteristics for $y \neq 0$

(vii) A particle executing a simple harmonic motion has acceleration 8 cm/sec^2 when it is at a distance 2 cm from the centre. The time period will be

(a) $\frac{2}{\pi}$ second

(b) $\frac{1}{\pi}$ second

(c) π second

(d) $\frac{\pi}{2}$ second

(vii) The necessary condition to exact the differential equation $M\,dx + N dy = 0$ is

(a) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$

(b) $\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$

(c) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(d) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(ix) The Residue of $f(z) = \frac{z^2}{(z-a)(z-b)(z-c)}$ at $z = \infty$ is

(a) 1

(b) -1

(c) e

(d) $-e$

(x) For any primal and its dual

(a) optimum value of the objective function is same.

(b) both primal and dual cannot be feasible.

(c) primal will have an optimum solution if and only if dual does too.

(d) all of these

PART – B

Attempt all questions.

2. If $\psi = \frac{a}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\gamma)^2}}$, show that $\nabla^2\psi = 0$. **(2)**

3. Show that the function $f(z) = e^{\frac{1}{z}}$ actually takes every values except zero an infinite number of times in the neighbourhood of $z = 0$. **(2)**

4. If $f(x) = \ln x$, find the value of $f'(2.0)$ from the given table **(2)**

i	1	2	3
x_i	2.0	2.2	2.6
f_i	0.69315	0.78846	0.95551

5. Find the complementary function of $x^2 y'' + 3xy' + y = (1-x)^{-2}$ **(2)**

6. Calculate $\int_2^{10} \frac{dx}{1+x}$ (up to 3 decimal places) by dividing the range into 8 equal parts by Simpson's $\frac{1}{3}$ rule. **(2)**

7. Find particular equation of the partial differential equation $r - 2s + t = 12xy$. **(2)**

8. Show that the set $X = \{(x_1, x_2) \mid x_1^2 + x_2^2 \le 4\}$ is a convex set. **(2)**

9. Use graphical method to solve the linear programming problem **(2)**

Maximize $z = 2x_1 + 4x_2$, subject to the constraints:

$x_1 + 2x_2 \le 5; x_1 + x_2 \le 4;\ x_1, x_2 \ge 0.$

10. Using Newton-Raphson method, find the root correct to 2 decimal places of the equation.

$x^3 - 5x + 3 = 0.$ **(2)**

11. Using Cauchy residue theorem evaluate

$\int_C \frac{z}{z^4 - 1} dz, C : |z| = 2.$ **(2)**

PART – C

Attempt any 6 (six) questions.

12. Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. **(10)**

13. Show that the equations $xp - yq = x,\ x^2 p + q = xz$ are compatible and hence solve it. **(10)**

14. Find the complete integrals of the PDE $z^2 = pqxy$. **(10)**

15. Apply Langrange's formula to interpolate $f(1.50)$ using the following values of the function $f(x)$

x	1.00	1.20	1.40	1.60	1.80	2.00
$f(x)$	0.2420	0.1942	0.1497	0.1100	0.0790	0.0540

(10)

16. Solve the following linear programming problem by simplex method. **(10)**

Maximize $z = 8x_1 + 7x_2 - 2x_3$

Subject to the constraints

$x_1 + 2x_2 + 2x_3 \leq 12$

$2x_1 + x_2 - 2x_3 \leq 12$

$x_1, x_2, x_3 \geq 0.$

17. Solve the differential equation **(10)**

$$\frac{dy}{dx} + xy = 0,\ y(0) = 1$$

from $x = 0$ *to* $x = 0.25$ taking $h = 0.05$ using Predictor - Corrector method.

18. Two materials A and B are required to construct tables and book cases. For one table 12 units of A and 16 units of B are needed while for a book case 16 units of A and 8 units of B are required. The profit on book case is Rs 25 and Rs 20 on table. 100 units of material A and 80 units of B are available. How many book cases and tables be produced to have maximum profit.

Formulate this as a LPP and solve by simplex method. **(10)**

19. Prove that every non constant polynomial over $\mathbb{C}$ (the field of complex numbers) has a root in $\mathbb{C}$. **(10)**

20. Solve $\frac{d^4y}{dx^4} - 16y = x^4 + \sin x.$ **(10)**

21. Use residues to evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$ **(10)**

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