CSM : 14

MATHEMATICS
PAPER - II

Time Allowed : 3 hours | Full Marks : 100

Marks for each question is indicated against it.

Attempt __any 5 (five)__ questions taking not more than 3 (three) questions from each Part.

PART A

1. (a) If $o(G)=p^2$, where p is a prime and G a group, show that G is abelian. **(7)**

(b) If f is a homomorphism of a group G onto a group G' and g is a homomorphism of G' onto a group G'', show that $g \circ f$ is a homomorphism of G onto G''. Also show that the kernel of f is a subgroup of that of $g \circ f$. **(4+4=8)**

(c) Show that the set of all real numbers of the form $x+y\sqrt{2}$, where x and y are integers, is an integral domain but not a field with respect to usual addition and multiplication. **(5)**

2. (a) State and prove the first mean value theorem for Riemann integrals. **(10)**

(b) If $x(r,\theta,\phi)=r\cos\theta\sin\phi$, $y(r,\theta,\phi)=r\sin\theta\sin\phi$, $z=r\cos\phi$, show that

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}=-r^2\sin\phi.$$ **(10)**

3. (a) Show that the complex function defined by $f(z)=\begin{cases} e^{-z^{-4}}, & z\neq 0 \\ 0, & z=0 \end{cases}$ satisfies Cauchy Riemann equations at $z=0$ but is not analytic at that point. **(10)**

(b) Expand the complex function $f(z)=\dfrac{1}{(z+1)(z+3)}$ in Laurent series valid for

(i) $1<|z|<3$ **(5)**

(ii) $|z|>3$ **(5)**

4. (a) Use Simplex method to solve the following LPP **(10)**

Minimize $z = x_1 - 3x_2 + 2x_3$ subject to

$3x_1 + x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$

(b) Use Vogel's approximation method to find the optimal solution of the following transportation problem for minimizing cost: **(10)**

		Destination			
		1	2	3	Availability
Origin	1	10	9	8	8
	2	10	7	10	7
	3	11	9	7	9
	4	12	14	10	4
Requirement		10	10	8	28

PART B

5. (a) Find the complete solution of the differential equation:

$(D + -3D' - 2)^2 z = 2e^{2x} \sin(y + 3x)$, where $D = \frac{\partial}{\partial x}$ & $D' = \frac{\partial}{\partial y}$ **(10)**

(b) The vibrations of an elastic string is governed by the partial differential equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$.
The length of the string π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$. Find the deflection $u(x,t)$ of the vibrating string for $t > 0$. **(10)**

6. (a) Calculate the value of $\int_{1.2}^{1.6} \left(x + \frac{1}{x} \right) dx,$ correct upto four significant figures, taking four sub-intervals by **(5+5=10)**

(i) Simpson's one third rule.

(ii) Trapezoidal rule.

(b) Find the value of y(0.2) by 4th order Runge–Kutta method which is correct to four decimal places, where $\frac{dy}{dx} = y^2 - x^2$, $y(0) = 1$ taking $h = 0.1$ **(10)**

7. (a) What are logic gates? Draw the block diagram of a 3-input XOR gate. Give its truth table. **(6)**

(b) Draw the flowchart for 1+2+3+.........+*n* **(4)**

(c) Convert the following binary numbers to octal form: **(5+5=10)**

(i) $(101.1001)_2$

(ii) $(0.1011101)_2$

8. (a) Derive the Hamilton's equation for a simple pendulum. **(10)**

(b) Show that the velocity field $u(x, y) = \frac{A(x^2 - y^2)}{(x^2 + y^2)^2}$, $v(x, y) = \frac{2Axy}{(x^2 + y^2)^2}$, $w = 0$, satisfies the equation of motion for inviscid incompressible flow. Determine the pressure associated with the velocity field. **(10)**

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