MIZORAM PUBLIC SERVICE COMMISSION

GENERAL COMPETITIVE EXAMINATIONS FOR RECRUITMENT TO THE POST OF JUNIOR GRADE OF MIZORAM PLANNING, ECONOMICS & STATISTICAL SERVICE UNDER PLANNING & PROGRAMME IMPLEMENTATION DEPARTMENT.

JANUARY, 2020

MATHEMATICS PAPER-II

Time Allowed : 3 hours | Full Marks : 100

Marks for each question is indicated against it.

Directions (Question No. 1 - 3): Attempt any 2 (two) questions.

1. What do you mean by the auxiliary equation of a given homogeneous linear differential equation. Solve the Differential equation $\frac{d^2y}{dx^2}+2\alpha\frac{dy}{dx}+\left(\alpha^2+\pi^2\right)y=0$ with the initial value $y(0)=3,\ y'(0)=-3\alpha$. **(3+7=10)**

2 (a) Solve $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-1\right)y=-3e^{x^2}\sin 2x$. **(5)**

(b) Using the method of variation of parameter, solve the differential equation $(x-1)y_2-xy_1+y=(x-1)^2$. **(5)**

3. (a) Solve $\frac{x\,dx}{z^2-2yz-y^2}=\frac{dy}{y+z}=\frac{dz}{y-z}$. **(3)**

(b) Solve $e^{3x}(p-1)+p^3e^{2y}=0$. **(3)**

(c) Solve $x^2y_2-2x(x+1)y_1+2(x+1)y=x^3$. **(4)**

Directions (Question No. 4 - 6): Attempt any 2 (two) questions.

4. Solve the equation $z(qs-pt)=pq^2$ by Monge's method. **(10)**

5. (a) Find the integral surface of the linear p. d. e.

$x\left(y^2+z\right)p-y\left(x^2+z\right)q=\left(x^2-y^2\right)z$. **(4)**

(b) Find the complete solution of the p.d.e. $x^2p^2+y^2q^2=z^2$. **(3)**

(c) Find the complete integral of the p.d.e. $pxy+pq+qy=yz$. **(3)**

6. (a) Reduce $\frac{\partial^2 z}{\partial x^2}+2\left(\frac{\partial^2 z}{\partial x\,\partial y}\right)+\frac{\partial^2 z}{\partial y^2}=0$ to canonical form and hence solve it. **(3)**

(b) Solve $4r-4s+t=16\log(x-2y)$. **(3)**

(c) Solve $t-r\sec^4 y=2q\tan y$. **(4)**

Directions (Question No. 7 - 9): Attempt any 2 (two) questions.

7. Using, Cauchy's Residue Theorem show that $\int_0^\infty \frac{1}{(z^2+a^2)^2}dx = \frac{\pi}{4a^3}$, where $a > 0$. **(10)**

8. (a) For what value of the z the function defined by the following equation ceases to be analytical $z = \log\rho + i\varphi$, where $w = \rho(\cos\varphi + i\sin\varphi)$. **(4)**

(b) If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = \frac{e^y - \cos x + \sin x}{\cosh x - \cos x}$.

Find $f(z)$ subject to the condition $f\left(\frac{\pi}{2}\right) = \frac{3-i}{3}$. **(3)**

(c) Expand $\frac{1}{z(z^2-3z+2)}$ in the region $1 < |z| < 2$. **(3)**

9. (a) Prove that $\cosh\left(z+\frac{1}{z}\right) = a_0 + \sum_1^\infty a_n\left(z^n + \frac{1}{z^n}\right)$, where $a_n = \frac{1}{2\pi}\int_0^{2\pi}\cos n\theta\cosh(2\cos\theta)\,d\theta$.

(3)

(b) What kind of singularities the following function have: **(4)**

(i) $\frac{1}{\sin z - \cos z}$ $at\ z = \frac{\pi}{4}$,

(ii) $z\cos ec\,z\ \ at\ \ z = \infty$.

(c) By using Residue theorem, Prove that $\int_0^{2\pi}\frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, $a > b > 0$. **(3)**

Directions (Question No. 10 - 12): Attempt any 2 (two) questions.

10. Apply Lagrange's formula to find a root of the equation $f(x) = 0$ when $f(30) = -30$, $f(34) = -13$, $f(38) = 3$, $f(42) = 18$. **(10)**

11. (a) Compute $f(0.5)$ from the following table **(3)**

x	0	1	2	3
$f(x)$	1	2	11	34

(b) Given the following value of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
0	1	2
1	2	2
2	9	14

Calculate the value of $f(1.1)$. **(4)**

(c) Evaluate $\int_0^1 x^3 dx$, by Trapezoidal Rule, with $n = 5$. **(3)**

12. (a) Compute the value of x for $y = 0.6742$ from the table **(5)**

x	3.5	4	4.8	5.6
$f(x)$	0.5441	0.6020	0.6812	0.7482

(b) Using Euler's method solve $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. Find $y(0.1)$. **(5)**

Directions (Question No. 13 - 15): Attempt any 2 (two) questions.

13. Solve the following linear programming problem in standard form using the simplex method:
Maximize $z = 8x_1 + 9x_2 + 5x_3$ subjected to

$x_1 + x_2 + 2x_3 \le 2$

$2x_1 + 3x_2 + 4x_3 \le 3$

$6x_1 + 6x_2 + 2x_3 \le 8$

$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$ **(10)**

14. (a) For the following L. P. P.

Max. $z = (3 - 6\lambda)x_1 + (2 - 2\lambda)x_2 + (5 + 5\lambda)x_3$

Subject to $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

and $x_1, x_2, x_3 \ge 0$.

Find the range of λ over which the solution remains basic feasible and optimal. **(5)**

(b) Solve the following transportation problem **(5)**

		To			Supply
		1	**2**	**3**	
From	**1**	2	7	4	5
	2	3	3	1	8
	3	5	4	7	7
	4	1	6	2	14
Demand		7	9	18	34

15. (a) Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is known and is given in the following table.

		I	**II**	**III**	**IV**	**V**
Man	**1**	2	9	2	7	1
	2	6	8	7	6	1
	3	4	6	5	3	1
	4	4	2	7	3	1
	5	5	3	9	5	1

Find the assignment of men to jobs that will minimize the total time taken. **(5)**

(b) Solve the L. P. P.

Max. $z = 6x_1 - 2x_2$

s.t. $2x_1 - x_2 \le 2$

$x_1 \le 4$

$x_1 + x_2 \ge 0$. **(5)**

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