TECHNICAL PAPER-I

Time Allowed : 3 hours
Full Marks : 150

Attempt all questions.

All questions carry equal marks of 2 each.

1. The unit digit in $7^{105}$ is
   (a) 9  (b) 7  (c) 3  (d) 1

2. Between 100 and 300, how many numbers are divisible by 7?
   (a) 14  (b) 21  (c) 28  (d) 35

3. $\sqrt{2500} + \sqrt{961} = (?)^2$
   (a) 3  (b) 9  (c) 81  (d) None of these

4. If $\frac{x}{y} = \frac{1}{3}$, then $\frac{x^2 + y^2}{x^2 - y^2} = ?$
   (a) $\frac{-10}{9}$  (b) $\frac{-5}{4}$
   (c) $\frac{5}{4}$  (d) $\frac{-5}{3}$

5. The mean of 100 items was found to be 30. If at the time of calculation, two items were wrongly taken as 32 and 12 instead of 23 and 11, the correct mean is:
   (a) 29.4  (b) 29.5  (c) 29.8  (d) 29.9

6. In a Zumba class, the average age of all the members was 43.5 years, 10 members left the class and 6 new members joined. If the average age increased by 2 years and the total age decreased by 110, what was the number of members in the class initially?
   (a) 34  (b) 36  (c) 32  (d) 40
7. What will come in place of the question marks (?).

\[(0.64)^2 \div (0.512)^3 \times (0.8)^4 = (0.8)^{?}\]

(a) 0  (b) 1  (c) 4  (d) 8

8. 10 men can finish a piece of work in 10 days, whereas it takes 12 women to finish it in 10 days. If 15 men and 6 women undertake to complete the work, how many days will they take to complete it?

(a) 2  (b) 4  (c) 5  (d) 7

9. A cistern has two pipes. One can fill it with water in 8 hrs and other can empty it in 5 hrs. In how many hrs will the cistern be emptied if both the pipes are opened together when 3/4 of the cistern is already full of water?

(a) 3 \frac{1}{3} hrs  (b) 6 hrs  (c) 10 hrs  (d) 13 \frac{1}{3} hrs

10. How much water must be added to a bucket which contains 40 litres of milk at the cost price of Rs 3.50 per litre so that the cost of milk reduces to Rs 2 per litre?

(a) 30 litres  (b) 35 litres  (c) 38 litres  (d) 40 litres

11. A reduction of 20% in the price of rice enables a person to buy 3.5 kg more rice for Rs 385. The original price of rice is:

(a) Rs 26.50  (b) Rs 27.50  (c) Rs 28.50  (d) Rs 29.50

12. A man sold two steel chairs for Rs 500 each. On one, he gains 20% and the other, he loses 12%. How much does he gain or lose in the whole transaction?

(a) 1.5% gain  (b) 2% gain  (c) 1.5% loss  (d) 2% loss

13. A fan is listed at Rs 150 with a discount of 20%. What additional discount must be offered to the customer to bring the net price to 108?

(a) 10%  (b) 12%  (c) 13%  (d) 15%

14. What number has to be added to each term 3 : 5 to make the ratio 5 : 6?

(a) 5  (b) 6  (c) 7  (d) 12

15. Value of machine depreciates @ 7% p.a. If its present value is Rs 23,000 then its value after 2 years will be

(a) Rs 19,890  (b) Rs 19,892.70  (c) Rs 19,850  (d) Rs 19,672
16. Choose the missing terms out of the given alternatives: GH, JL, NQ, SW, YD, ?
   (a) EJ    (b) FJ
   (c) EL    (d) FL

17. Choose the missing terms out of the given alternatives:
   Newspaper : Press : : Cloth : ?
   (a) Mill    (b) Fibre
   (c) Textile (d) Tailor

18. Choose the word which is least like the other words in the group.
   (a) January    (b) May
   (c) July       (d) November

19. If ROAST is coded as PQYUR in a certain language, then how will SLOPPY be coded in that language?
   (a) MRNAQN    (b) NRMNQA
   (c) QNMRNA    (d) RANNMQ

20. P, Q, R and S are playing a game of carrom. P, R and S, Q are partners. S is to the right of R who is facing west. Then Q is facing
   (a) North    (b) South
   (c) East      (d) West

21. Study the diagram below and identify the region representing youth who are employed but not educated.

22. Find the number of triangles in the given figure.

   (a) 10    (b) 19
   (c) 21    (d) 23
23. Find out which of the answer figures (a), (b), (c) and (d) completes the figure matrix?

![Figure Matrix](image)

24. Select a figure from amongst the four alternatives, which when placed in the blank space of figure X.

![Figure X](image)

(a) ![Figure A](image)  (b) ![Figure B](image)  
(c) ![Figure C](image)  (d) ![Figure D](image)

25. The position of how many letters in the word BRAKES remains unchanged when they are arranged in alphabetical order?

(a) One  (b) Two  
(c) Three  (d) More than three

26. The values of \(k\) for which the quadratic equation \(kx^2 + 1 = kx + 3x - 11x^2\) has real and equal roots are:

(a) -11, -3  (b) 5, 7  
(c) 5, -7  (d) -5, 7
27. For the equation \(3x^2 + px + 3 = 0, p > 0\), if one of the root is square of the other, then \(p =\)
(a) -6, 3  
(b) 6, -3  
(c) 1, -3  
(d) -1, 3

28. If the roots of the equation \(5x^2 - 7x + k = 0\) are reciprocal to each other, then the value of \(k\) is
(a) \(\frac{1}{5}\)  
(b) 5  
(c) 7  
(d) \(\frac{1}{7}\)

29. If \(x = \log_a bc, y = \log_b ca\) and \(z = \log_c ab\), then \(\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} =\)
(a) \(x + y + z\)  
(b) \(ab + bc + ca\)  
(c) \(abc\)  
(d) 1

30. An equation of the form \(ax + by + c = 0\). Where, \(a \neq 0, b \neq 0\) and \(c = 0\) represents a straight line which passes through
(a) (2,4)  
(b) (0,0)  
(c) (3,2)  
(d) (2,3)

31. If A and B are any two sets, then \((A \cup B) - (A \cap B) =\)
(a) \((A - B) \cup (B - A)\)  
(b) \((A - B) \cap (B - A)\)  
(c) \(A - B\)  
(d) \(B - A\)

32. If \(\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}\), then \(x^y y^z z^x = ? (x, y, z > 0)\)
(a) \(e\)  
(b) 1  
(c) \(e^e\)  
(d) 0

33. Let \(P(n)\) be the statement \(2^n > n!\), where \(n\) is a natural number, then \(P(n)\) is true for
(a) all \(n\)  
(b) all \(n > 1\)  
(c) all \(n > 2\)  
(d) all \(n > 3\)

34. If a polynomial \(f(x)\) is divided by \(ax - b\), then the remainder is
(a) \(f\left(-\frac{b}{a}\right)\)  
(b) \(f\left(-\frac{b}{a}\right)\)  
(c) \(f\left(-\frac{a}{b}\right)\)  
(d) \(f\left(-\frac{a}{b}\right)\)
35. An equilateral triangle has side $2\sqrt{3}$ cm. The radius of its circumcircle will be
   (a) 2 cm (b) $\sqrt{3}$ cm (c) 3 cm (d) 4 cm

36. The sides of two similar triangles are in the ratio 4 : 9. The areas of these triangles are in the ratio
   (a) 2 : 3 (b) 4 : 9 (c) 81 : 16 (d) 16 : 81

37. If $\tan^2 \theta = (1 - e^2)$, then $\sec \theta + \tan^3 \theta \cos \theta =$
   (a) $(1 - e^2)^{\frac{3}{2}}$ (b) $(1 + e^2)^{\frac{3}{2}}$
   (c) $(2 - e^3)^{\frac{3}{2}}$ (d) $(2 + e^3)^{\frac{3}{2}}$

38. If $A$ lies in the third quadrant and $3 \tan A - 4 = 0$, then $5 \sin 2A + 3 \sin A + 4 \cos A =$
   (a) 0 (b) $-\frac{24}{5}$
   (c) $\frac{24}{5}$ (d) $\frac{48}{5}$

39. The angle of elevation of the top of the tower from two points at a distance ‘a’ and ‘b’ from the base
   and in the same straight line with it are complementary. The height of the tower is
   (a) $a + b$ (b) $\sqrt{ab}$
   (c) $a\sqrt{b}$ (d) $b\sqrt{a}$

40. A man on the top of a rock lying on a sea shore observes a boat coming towards it. If it takes 10
   minutes for the angle of depression to change from 30° to 60°, how soon will the boat reach the
   shore?
   (a) 5 minutes (b) 10 minutes
   (c) 15 minutes (d) 20 minutes

41. If the points $(a,0),(0,b)$ and $(1,1)$ are collinear, then $\left(\frac{a+b}{ab}\right) =$
   (a) 2 (b) $\sqrt{2}$
   (c) -1 (d) 1

42. The point which divides the line segment joining the points $(3,4)$ and $(7,-6)$ in the ratio 1 : 2 internally
   lies in the
   (a) I quadrant (b) II quadrant
   (c) III quadrant (d) IV quadrant

43. What is the perimeter of the triangle formed by the points $(0,0),(1,0)$ and $(0,1)$?
   (a) $\sqrt{2}$ (b) 2
   (c) $2 + \sqrt{2}$ (d) $2 - \sqrt{2}$
44. The graph of $3x - 5y = -8$ and $3x + 5y = 32$ intersect at the point $(p, q)$, then $p - q =$
(a) 0  (b) 1  (c) 2  (d) 3

45. The minimum value of $\sin^2 \theta + \cos^4 \theta$ is
(a) $\frac{1}{2}$  (b) $\frac{3}{2}$  (c) $\frac{2}{3}$  (d) $\frac{3}{4}$

46. If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(AB^c) = 0.5$, then
$P(B / A \cup B^c) =$
(a) 0.25  (b) 0.26  (c) 0.27  (d) 0.28

47. If $E_1$ and $E_2$ are mutually exclusive events, then
(a) $P(E_1) + P(E_2) \leq 1$  (b) $P(E_1) + P(E_2) = 1$
(c) $P(E_1) + P(E_2) \geq 1$  (d) $P(E_1) + P(E_2) < 1$

48. If $A_1, A_2, \ldots, A_8$ are independent events such that $P(A_i) = \frac{1}{i+1}, 1 \leq i \leq 8$, then the probability that none of the events occurs is
(a) $\frac{1}{2}$  (b) $\frac{1}{3}$
(c) $\frac{1}{9}$  (d) $\frac{2}{9}$

49. For a normal distribution, if the mean is $M$, mode is $M_o$ and median is $M_d$, then
(a) $M > M_d > M_o$  (b) $M > M_o > M_d$
(c) $M = M_o \neq M_d$  (d) $M = M_o = M_d$

50. For any two events $E_1$ and $E_2$, then $P((E_1 \cup E_2) \cap (E_1^c) \cap (E_2^c)) =$
(a) $\frac{1}{4}$  (b) $\leq \frac{1}{4}$
(c) $\geq \frac{1}{2}$  (d) 1

51. If the mean of the binomial distribution is 25, then its standard deviation lies in the interval
(a) [5, 25)  (b) (5, 25]
(c) [0, 5)  (d) (0, 5]
52. If there are 3 children in a family, then the probability that there is one girl in the family is

(a) $\frac{2}{3}$  
(b) $\frac{1}{3}$  
(c) $\frac{3}{28}$  
(d) $\frac{3}{8}$

53. If $A$ and $B$ are two events such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\overline{A} / \overline{B}) =$

(a) $1 - \frac{P(A \cup B)}{P(B)}$  
(b) $1 - \frac{P(\overline{A} \cup \overline{B})}{P(B)}$  
(c) $1 - P(A / B)$  
(d) $1 - \frac{P(A)}{P(B)}$

54. Suppose $X$ is a binomial variate $B(5, p)$ and $P(X=2) = P(X = 3)$, then $p =$

(a) $\frac{1}{5}$  
(b) $\frac{1}{3}$  
(c) $\frac{1}{2}$  
(d) $\frac{1}{4}$

55. A box contains 50 tickets numbered 1,2,3,..............,50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_3 = 30$ is

(a) $\frac{20}{50} \cdot \frac{30}{50}$  
(b) $\frac{20}{50} \cdot \frac{20}{50}$  
(c) $\frac{20}{50} \cdot \frac{29}{50}$  
(d) $\frac{20}{50} \cdot \frac{20}{50}$

56. If $b_{yx}$ and $b_{xy}$ are regression coefficient of Y and X and X on Y respectively, then which of the following statements is true ?

(a) $b_{yx} = 1.5, b_{yx} = 0.6$  
(b) $b_{yx} = 1.5, b_{xy} = 1.4$  
(c) $b_{yx} = 1.5, b_{yx} = 0.9$  
(d) $b_{yx} = 1.5, b_{yx} = 0.8$

57. The standard deviation of 25 numbers is 40. If each of the numbers is increased by 5, then the new standard deviation will be

(a) 46  
(b) 45  
(c) 44  
(d) 40

58. If the two lines of regression of a bivariate distribution coincide, then the correlation coefficient $\rho$ satisfies

(a) $\rho = 1 or -1$  
(b) $\rho = 0$  
(c) $\rho < 0$  
(d) $\rho > 0$
59. If the variance of \( x \) is 9 and regression equations are \( 4x - 5y + 33 = 0 \) and \( 20x - 9y - 10 = 0 \), then the coefficient of correlation between \( x \) and \( y \) and the variance of \( y \) are respectively:
   (a) 0.6, 4  
   (b) 0.6, 16  
   (c) 0.3, 4  
   (d) 0.3, 16

60. The two lines of regression for a certain bivariate distribution \((x,y)\) are (I) \( x + 2y = 3 \), (II) \( 2x - y = 4 \) then the regression line of \( x \) on \( y \) is
   (a) I  
   (b) II  
   (c) any of I and II  
   (d) None of these

61. The function \( f(x) = ax + b \) is strictly increasing for all \( x \in \mathbb{R} \) if
   (a) \( a > 0 \)  
   (b) \( a < 0 \)  
   (c) \( a = 0 \)  
   (d) None of these

62. The curve \( y = x^\frac{1}{3} \) has at \((0, 0)\)
   (a) a horizontal tangent  
   (b) a vertical tangent  
   (c) oblique tangent  
   (d) no tangent

63. \( \lim_{x \to a} \frac{a^{\frac{x}{3}} - 1}{\sqrt[3]{a} - 1} = \)
   (a) \( a \log 2 \)  
   (b) \( \frac{1}{2} \log a \)  
   (c) \( 2 \log a \)  
   (d) \( \frac{1}{a} \log 2 \)

64. Consider the function
   \[
   f(x) = \begin{cases} 
   x^2 & , x \neq 0 \\
   |x| & , x = 0
   \end{cases}
   \]
   then
   (a) \( f(x) \) is discontinuous everywhere  
   (b) \( f'(x) \) exist at all \( x \) in \((-1,1)\)  
   (c) \( f'(x) \) exist at all \( x \) in \((-2,2)\)  
   (d) \( f(x) \) is continuous everywhere

65. Let \( f(x) \) and \( g(x) \) be differentiable functions for \( 0 \leq x \leq 1 \), such that \( f(0) = 2, g(0) = 0 \) , \( f(1) = 6 \). Let there exist a real number \( c \) in \((0, 1)\) such that \( f'(c) = 2g'(c) \), then the value of \( g(1) \) must be
   (a) 1  
   (b) 2  
   (c) -2  
   (d) -1
66. If the function \( f(x) = x^3 + \alpha x^2 + \beta x + 1 \) has a maximum value at \( x = 0 \) and minimum value at \( x = 1 \), then

(a) \( \alpha = 0, \beta = \frac{3}{2} \)  
(b) \( \alpha = 0, \beta = -\frac{3}{2} \)  
(c) \( \alpha = -\frac{3}{2}, \beta = 0 \)  
(d) \( \alpha = \frac{3}{2}, \beta = 0 \)

67. Rolle’s theorem holds for the function \( x^3 + bx^2 + cx, 1 \leq x \leq 2 \) at the point \( \frac{4}{3} \), the value of \( a \) and \( b \) are

(a) \( b = 8, c = -5 \)  
(b) \( b = 5, c = -8 \)  
(c) \( b = -5, c = -8 \)  
(d) \( b = -5, c = 8 \)

68. If \( ax^2 + 2hxy + by^2 = 1 \), then \( \frac{d^2 y}{dx^2} = \)

(a) \( \frac{h^2 + ab}{(hx + by)^3} \)  
(b) \( \frac{h^2 - ab}{(hx + by)^3} \)  
(c) \( \frac{h^2 - ab}{(hx + by)^3} \)  
(d) \( \frac{h^2 + ab}{(hx + by)^3} \)

69. The function \( f(x) = 2x^3 - 15x^2 + 36x + 4 \) is maximum at

(a) \( x = 3 \)  
(b) \( x = 0 \)  
(c) \( x = 4 \)  
(d) \( x = 2 \)

70. The derivative of \( f(x) = 3 |2 + x| \) at the point \( x = -3 \) is

(a) \(-3\)  
(b) \(3\)  
(c) \(0\)  
(d) not determined

71. If \( u = \sqrt{x^2 + y^2 + z^2} \), then \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = \)

(a) \(-u\)  
(b) \(u\)  
(c) \(2u\)  
(d) \(\frac{1}{u}\)

72. If \( u = \sin^{-1} \left( \frac{x^2 + y^2}{x + y} \right) \), then \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \)

(a) \(\sin u\)  
(b) \(\cos u\)  
(c) \(\tan u\)  
(d) \(\cot u\)
73. \[ \int 2^x \, dx = \]

(a) \( \frac{2^x}{(\log 2)^x} + C \) \hspace{1cm} (b) \( \frac{2^x}{(\log 2)^x} + C \)

(c) \( \frac{2^x}{\log_2 e} + C \) \hspace{1cm} (d) \( \frac{2^x}{\log_2 e} + C \)

74. If \( f(x) \) is odd function, then \[ \int_{-\pi/2}^{\pi/2} f(\cos x) \, dx = \]

(a) 0 \hspace{1cm} (b) \( \frac{\pi}{2} \int_{0}^{\pi/2} f(\cos x) \, dx \)

(c) \( 2 \int_{0}^{\pi/2} f(\sin x) \, dx \) \hspace{1cm} (d) \( \int_{0}^{\pi} f(\cos x) \, dx \)

75. The integral \[ \int_{-1}^{1} \frac{|x + 2|}{x + 2} \, dx = \]

(a) 0 \hspace{1cm} (b) 1

(c) 2 \hspace{1cm} (d) -1

* * * * * * *