MIZORAM PUBLIC SERVICE COMMISSION

TECHNICAL COMPETITIVE EXAMINATIONS FOR RECRUITMENT TO
INSPECTOR OF STATISTICS
UNDER PLANNING & PROGRAMME IMPLEMENTATION DEPARTMENT, FEBRUARY, 2017

PAPER - III
MATHEMATICS

Time Allowed : 3 hours | Full Marks : 100

Marks for each paper is marked against it.
Attempt all questions.

(OBJECTIVE) *Choose a correct answer from the given alternatives:* **(20×1=20)**

1. The number of surjection from A = { 1, 2, ..., n }, $n \geq 2$, onto B = { a, b } is

(a) ${}^{n}P_2$ (b) ${}^{n}C_2$

(c) $2^n - 2$ (d) $2^n - 1$

2. The number of positive roots of the equation $x^5 - 6x + 2 = 0$ is

(a) 5 (b) 2

(c) 3 (d) 4

3. The definite integral $\int_0^2 |x-1|\,dx$ has value

(a) 0 (b) 1

(c) 2 (d) does not exist

4. If $u = \log(x^2 + y^2),\ (x, y) \neq (0,0),\ then\ \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} =$

(a) e (b) 2

(c) 1 (d) 0

5. The angle between the pair of lines $ax^2 + 2hxy + by^2 = 0$ is given by

(a) $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}$ (b) $\tan\theta = \dfrac{\sqrt{h^2 - ab}}{a+b}$

(c) $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a-b}$ (d) $\tan\theta = \dfrac{\sqrt{h^2 - ab}}{a-b}$

6. If $\vec{a}$ and $\vec{b}$ are constant vectors, then

(a) $\nabla.(\vec{a}.\vec{b}) = \vec{a}$

(b) $\nabla.(\vec{a}.\vec{b}) = \vec{b}$

(c) $\nabla.(\vec{a}.\vec{b}) = \vec{0}$

(d) None of these.

7. The radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$ is

(a) 5

(b) 4

(c) 3

(d) 2

8. The integrating factor of the differential equation $ydx - xdy = 0$ is

(a) x^2

(b) x

(c) $\frac{1}{x}$

(d) $\frac{1}{x^2}$

9. The complete integral of the differential equation $p^2 = zq$ is

(a) $z = be^{ax+a^2y}$

(b) $z = be^{x+a^2y}$

(c) $z = be^{x+ay}$

(d) None of these

10. The position of a moving point at time t referred to rectangular axes is given by $x = a\cos\omega t,\ y = a\sin\omega t$. The path of the particle is a

(a) straight line

(b) circle of radius a

(c) cycloid

(d) parabola of latus rectum $4a$

11. The least velocity with which a body can be projected to have horizontal range R is

(a) $\sqrt{g/R}$

(b) $\sqrt{\mathrm{R/g}}$

(c) $\sqrt{gR}$

(d) None of these

12. If a body is in equilibrium, under the action of three equal non-parallel coplanar forces, then the angle between any two of these forces is

(a) 45°

(b) 60°

(c) 90°

(d) 120°

13. The centre of gravity of a uniform semi-circular lamina of radius a lies on the radius bisecting the lamina at a distance of

(a) $\frac{4a}{2\pi}$ from the centre

(b) $\frac{4a}{3\pi}$ from the centre

(c) $\frac{a}{3\pi}$ from the centre

(d) $\frac{a}{2\pi}$ from the centre

14. For any primal and its dual.

(a) optimum value of the objective function is same.

(b) both primal and dual cannot be feasible.

(c) primal will have an optimum solution if and only if dual does too.

(d) all the above

15. While solving an IPP, any non-integer variable in the solution is picked up to

(a) enter the solution (b) leave the solution

(c) obtain the cut constraint (d) all of the above

16. For maximization LPP, the objective function coefficient for an artificial variables is

(a) + M (b) - M

(c) + 1 (d) zero

17. The solution of transportation problem with m-sources and n-destinations is feasible, if the number of allocations are

(a) m + n - 1 (b) m + n + 1

(c) m + n (d) m x n

18. The real root of the equation $\cos x = 3x - 1$ lies between

(a) 0 and 1 (b) $0 \; and \; \pi$

(c) $0 \; and \; 2\pi$ (d) $0 \; and \; \frac{\pi}{2}$

19. In the General quadrature formula, Simpson's 1/3rd Rule is obtained by putting

(a) $n = 1$ (b) $n = 2$

(c) $n = 2$ & 4 (d) $n = 4$

20. Suppose X is binomially distributed with parameters n and p. If $E[X] = 5$ and $Var[X] = 4$, then

(a) $n = 25 \;\; and \;\; p = \frac{1}{5}$ (b) $n = 20 \;\; and \;\; p = \frac{1}{5}$

(c) $n = 25 \;\; and \;\; p = \frac{2}{5}$ (d) $n = 20 \;\; and \;\; p = \frac{2}{5}$

(SHORT ANSWER) *Answer all questions:* **(10×2=20)**

21. Suppose X has a binomial distribution with parameters n and p. For what p is $Var[X]$ maximized if we assumed n is fixed.

22. If X is normally distributed with mean 2 and variance 1, show that $P[|X - 2| < 1] = P[-1 < Z < 1]$.

23. Prove that if for every element in a group G, $a^2 = e$ then G is abelian (e is the identity element of a group).

24. Find the equilibrium point if the supply and demand equations of a product are

$x_s = 2p - 8 \;\; and \;\; x_d = 300 - 2p$, respectively.

25. If the total cost function is given by $C = a + bx + cx^2$, where x is the quantity of output, show that

$$\frac{d}{dx}(AC) = \frac{1}{x}(MC - AC)$$

where MC and AC are marginal and average cost.

26. Show that $\Delta^n e^x = e^x(e^h - 1)^n$ (where Δ denote the difference operator).

27. Show that the differential equation $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$ is exact.

28. If a particle moves so that its normal acceleration is always zero. Then show that the path is a straight line.

29. If a particle moves in a plane with constant speed. Prove that its acceleration is perpendicular to its velocity.

30. Prove that if a body be on the point of sliding down an inclined plane under its own weight, the inclination of the plane is equal to the angle of friction.

(DESCRIPTIVE) *Answer* ***<u>any 3 (three)</u>*** *questions:* **(3×20=60)**

31. (a) A manufacturer determines that t employees will produce a total of x units of a product per day, where $x = 2t$. If the demand equation for the product is $p = -0.5x + 20$, determined the marginal revenue product when $t = 5$. Interpret your result. **(5)**

(b) If $lx + my = 1$ is normal to the parabola $y^2 = 4ax$, then show that $al^3 + 2alm^2 = m^2$. **(5)**

(c) If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$ **(5)**

(d) Evaluate $\int_C xyds$ along the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ lying in the first quadrant. **(5)**

32. (a) Prove that every subgroup of a cyclic group is cyclic **(5)**

(b) Prove that any two right cosets of a subgroup are either disjoint or identical. **(5)**

(c) Find the remainder when $x^5 - 3x^4 + 4x^2 + x + 4$ is divided by $(x+1)(x-2)$. **(5)**

(d) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$; form the equation whose roots are $\frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}, \frac{\alpha\beta}{\gamma}$. **(5)**

33. (a) Solve $x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$ **(10)**

(b) Apply Charpit's method to solve $(p^2 + q^2)y = qz$. **(10)**

34. (a) Find the greatest and least distance from the point (2, -1, 1) to the sphere. **(10)**

$x^2 + y^2 + z^2 - 8x + 4y - 6z + 4 = 0$

(b) A Coca-cola Company want to design a drinking Can with capacity of 330ml. In what dimension should you make so that a minimum material is used? **(10)**

35. (a) Using simplex method solve the given LPP **(10)**

Maximize $Z = 5x_1 + 4x_2 + 3x_3$

Subject to the constraints

$$3x_1 + 2x_2 + x_3 \leq 10$$
$$2x_1 + x_2 + 2x_3 \leq 12$$
$$x_1 + x_2 + 3x_3 \leq 15$$
$$x_1, x_2, x_3 \geq 0$$

(b) Three factories (S_1, S_2, S_3) ship goods to three ware house (D_1, D_2, D_3). The weekly factory factory suplies are 50, 40 and 60 units. The respective weekly demands are 20, 95 and 35 units. The associated transportation costs are : **(10)**

	D_1	D_2	D_3
S_1	6	4	1
S_2	3	8	7
S_3	4	4	2

Find the initial basic feasible solution by using

(i) Northwest - corner rule

(ii) Least - cost method

36. (a) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as poisson variate with mean 1.5. Calculate the proportion of days on which **(10)**

(i) neither car is used

(ii) some demand is refused ($e^{-1.5} = 0.2231$)

(b) A minimum height is to be prescribed for eligibility to government services such that 60% of the youngmen will have a fair chance of coming up to that standard. The heights of youngmen are normally distributed with mean $60.6''$ and s.d. $2.55''$ **(10)**

Determine the minimum specification.

(Given that if $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-t^2/2)dt$, then $f(-0.2533) = 0.6$)

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