# MIZORAM PUBLIC SERVICE COMMISSION

## General Competitive Examinations for Recruitment to the post of Junior Grade of Mizoram Planning, Economics & Statistical Service under Planning & Programme Implementation Department.

## January, 2020

## MATHEMATICS PAPER-I

Time Allowed : 3 hours　　　　Full Marks : 100

*Marks for each question is indicated against it.*

***Directions (Question No. 1 - 3): Attempt any 2 (two) questions.***

**1.** Define discontinuity of the first kind and second kind. Show that the function define by

$$f(x)=\begin{cases}1 & (x\, rational)\\ 0 & (x\, irrational)\end{cases}$$

has discontinuity of the second kind at every point. **(2+8=10)**

**2.** Show that the sequence of functions $\{f_n\}$ defined on $E$ converges uniformly on $E$ if and only if for every $\varepsilon > 0$ there exists an integer $N$ such that $m \geq N$, $n \geq N$, $x \in E$ implies $\left|f_n(x) - f_m(x)\right| \leq \varepsilon$ **(10)**

**3.** (a) If $z$ is the function of $x$ and $y$ and $x^x y^y z^z = c$ then show that $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ when $x = y = z$. **(4)**

(b) If $f(x,y) = x^2 + xy - y^2$, then find $f_{yx}(1,2)$ $f_{yx}(1,2)$ by using the definition of partial derivatives. **(3)**

(c) Obtain Taylor's expansion of $f(x,y) = \cos(x+y)$ at the point $(0,0)$ for $n = 3$. **(3)**

***Directions (Question No. 4 - 6): Attempt any 2 (two) questions.***

**4.** (a) A normal linear space $E$ is a Branch space if and only if every absolutely summable sequence in $E$ is summable in $E$. **(5)**

(b) Show that an additive and homogeneous operator $A$ be continuous, it is necessary and sufficient that it is bounded. **(5)**

**5.** Show that the norm in an inner product space satisfy parallelogram inequality. And hence show that the space ð$[a,b]$ of all continuous real-valued function defined on $[a,b]$ with norm defined by $\|x\| = \max_{t\in[a,b]} |x(t)|$ is not an inner product space **(4+6=10)**

**6.** (a) If $A$ is the bounded linear operator in Helbert space $H$, and $A^*$ is the adjoint of $A$, then show that

(i) $A^{**} = A$　　　(ii) $\left(A^*\right)^{-1} = \left(A^{-1}\right)^*$. **(4)**

(b) If $P$ is projection in Helbert space then show that $P$ is a self-adjoint operator with its norm equal to one and $P$ satisfies $P^2 = P$. **(6)**

***Directions (Question No. 7 - 9): Attempt any 2 (two) questions.***

**7.** What is the difference between homomorphism and isomorphism? Show that a homomorphism $\phi$ of $G$ into $\bar{G}$ with kernel $K$ is an isomorphism if and only if $K = \{e\}$. **(2+8=10)**

**8.** Define Euclidean ring. Let $R$ be an Euclidean ring, then show that any two elements $a$ and $b$ in $R$ have a greatest common divisor $d$. Moreover show that $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$. **(2+8=10)**

**9.** Let $S = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \mid a, b \in Z \right\}$ **(10)**

(i) Check that $S$ is a subring of $M_2(R)$ and it is a commutative ring with identity.

(ii) Is $S$ an ideal of $M_2(R)$? Justify your answer.

(iii) Is $S$ an integral domain ? Justify your answer.

(iv) Find all the units of the ring $S$.

(v) Check whether $T = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \mid a, b \in Z, 2 \mid a \right\}$ is an ideal of $S$.

***Directions (Question No. 10 - 12): Attempt any 2 (two) questions.***

**10.** (a) Let $X = \{a, b, c\}$ and $T = \{\varphi, x, \{b\}, \{a, c\}\}$. Find interior, closure and set of all limit points of the set $\{a, b\}$. **(4)**

(b) In a topological space $\{X, T\}$ if $A \subset X$, then prove that

(i) $\bar{A} = A^0 \cup b(A)$, (ii) $A^0 = A - b(A)$. **(6)**

**11.** (a) Define base of a topology.

For $X = \{1, 2, 3, 4\}$ and $A = \{\{1, 2\}, \{2, 4\}, \{3\}\}$ determine the topology on $X$ generated by the elements of $A$ and hence determine the base for this topology. **(5)**

(b) Show that a mapping $f : X \to Y$ is continuous if and only if $\overline{f^{-1}(B)} \subset f^{-1}(\bar{B})$, for any $B \subset Y$. **(5)**

**12.** Define Hausdorff space. Show that that every compact subset of a Hausdorff space is closed. **(2+8=10)**

***Directions (Question No. 13 - 15): Attempt any 2 (two) questions.***

**13.** Derive the Lagrange's equations for a conservative and holonomic dynamical system. **(10)**

**14.** Derive Hamilton's equations of motion for a simple pendulum. **(10)**

**15.** (a) Setup the Lagrangian for a simple pendulum and obtain an equation describing its motion. **(5)**

(b) Obtain Lagrange's equation of motion for a system under finite forces. **(5)**

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