CSM : 15

MATHEMATICS

PAPER - II

Time Allowed : 3 hours Full Marks : 100

Marks for each question is indicated against it.

Attempt any 5 (five) questions taking not more than 3 (three) questions from each Part.

PART A

1. (a) Prove that a subgroup N of a G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G. **(8)**

(b) If S is an ideal of a ring R and T be any subring of R. Show that $(S+T)/S \cong T/S(S \cap T)$ **(12)**

2. (a) If H is a homogeneous function of x, y, z of degree n. Prove that

$$x\frac{\partial H}{\partial x}+y\frac{\partial H}{\partial y}+z\frac{\partial H}{\partial z}=nz.$$ **(6)**

(b) Show that the function $f:(0,1]\to \mathbb{R}$ defined by $f(x)=\frac{1}{x}$ is not uniformly continuous on $(0,1]$. **(7)**

(c) Find the maxima and minima of the function $f:\mathbb{R}^2\to \mathbb{R}$ defined by

$$f(x,y)=x^3+y^3-3x-12y+20.$$ **(7)**

3. (a) Prove that for $a>1$, $\int_0^{\pi}\frac{d\theta}{a+\cos\theta}=\frac{\pi}{\sqrt{a^2-1}}$ **(10)**

(b) Use method of contour integration to prove that

$$\int_0^{2\pi}\frac{d\theta}{1+a^2-2a\cos\theta}=\frac{2\pi}{1-a^2},\ 0\le a<1.$$ **(10)**

4. (a) Solve the following LPP by simplex method

Maximize $z = 4x_1 + 7x_2$ subject to

$$2x_1 + x_2 \le 10$$
$$x_1 + x_2 \le 6$$
$$x_1 + 2x_2 \le 10$$
$$x_1, x_2 \ge 0$$

(10)

(b) Find the optimal solution and the corresponding cost of transportation in the following transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

(10)

PART B

5. (a) Reduce the equation $y^2\left(\frac{\partial^2 z}{\partial x^2}\right) - 2xy\left(\frac{\partial^2 z}{\partial x \partial y}\right) + x^2\left(\frac{\partial^2 z}{\partial y^2}\right) = \left(\frac{y^2}{x}\right)\left(\frac{\partial z}{\partial x}\right) + \left(\frac{x^2}{y}\right)\left(\frac{\partial z}{\partial y}\right)$

to Canonical form and hence solve it. **(10)**

(b) By method of separation of variables solve the equation

$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ if $u(x,0) = 4x - \frac{1}{2}x^2$. **(10)**

6. (a) Compute $y(0.1)$ by Runge-Kutta method of fourth order for the differential equation

$\frac{dy}{dx} = xy + y^2$, $y(0) = 1$. **(7)**

(b) Use Gauss elimination method to solve the following system of equations:

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

(7)

(c) Find a real root of $x^3 - x - 4 = 0$ by bisection method correct upto four significant digits. **(6)**

7. (a) Write the following Boolean functions in their respective conjunctive normal form:

(i) $f(x,y,z) = (xy' + xz)' + x'$ **(5)**

(ii) $f(x,y,z) = xyz + x'yz + xy'z' + x'yz'$ **(5)**

(b) Convert the following numbers from the given bases to the bases indicated: **(4×2=8)**

(i) $(1.5)_{10} = (?)_2$

(ii) $(DEAF)_{16} = (?)_2$

(iii) $(00001000)_2 = (?)_{16}$

(iv) $(76543210)_8 = (?)_{16}$

(c) Write an algorithm for Simpson's $\frac{1}{3}$ rule. **(2)**

8. (a) Use Hamilton's equations to find the equations of motion of a projectile in space. **(10)**

(b) Use Lagrange's equations to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis. **(10)**

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