CSM : 22

MATHEMATICS
PAPER - II

Time Allowed : 3 hours   Full Marks : 100

Marks for each question is indicated against it.
Attempt any 5 (five) questions taking not more than 3 (three) questions from each Part.

PART - A

1. (a) Prove that a field is an integral domain. Is the converse true? Justify your answer. **(10)**

(b) Let R be a commulative ring and S an ideal of R. Prove that the ring of residue classes R/S is an integral domain if and only if S is a prime ideal. **(10)**

2. (a) Show that the sequence of functions $\{f_n(x)\}$, where $f_n(x) = \mathrm{e}^{-\mathrm{nx}}$ is pointwise but not uniformly convergent in $[0,\infty)$. Also show that the convergence is uniform in $[k,\infty)$, $k > 0$. **(10)**

(b) Check the convergence of the integral **(10)**

$$\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$$

3. (a) If $f(z)$ is an analytical function of z and $f'(z)$ is continuous at each point within and on a closed contour C, then prove that $\int_C f(z)dz = 0$. **(8)**

(b) Apply the calculus of residues to prove that **(7)**

$$\int_0^{\pi} \frac{1+2\cos\theta}{(5+4\cos\theta)} d\theta = 0$$

(c) If $f(z)$ has a pole at $z = a$, then prove $|f(z)| \to \infty \; as \; x \to a$ **(5)**

4. (a) Solve, by Simplex Method, the following L.P.P.: **(12)**

$Max\, z = 3x_1 + 5x_2 + 4x_3$ subject to the constraints:

$2x_1 + 3x_2 \le 8$

$2x_2 + 5x_3 \le 10$

$3x_1 + 2x_2 + 4x_3 \le 15$

and $x_1, x_2, x_3 \ge 0$

(b) Use duality to solve the LPP: **(8)**

$Min.z = 3x_1 + x_2$ subject to

$2x_1 + 3x_2 \geq 2$

$x_1 + x_2 \geq 1$

and $x_1, x_2 \geq 0$

PART - B

5. (a) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x+y+z, x^2+y^2-z^2) = 0$. **(7)**

(b) Solve: $(mz-ny)dx + (nx-lz)dy = ly - mx$. **(6)**

(c) Solve $sy - 2xr - 2p = 6xy$ **(7)**

6. (a) Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ **(10)**

by taking 7 ordinates at $x = 0, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}$.

Here $h = \frac{1}{6}$.

(b) Find by Newton-Raphson's method the real root of $2x - 3sinx - 5 = 0$. **(10)**

7. (a) Show that **(4)**

(i) $(176)_8=(126)_{10}$ (ii) $(17AB)_{10}=(6059)_{10}$

(b) Convert the following binary numbers to octal and hexadecimal form: **(12)**

(i) 110101 (ii) 101111001

(iii) 100110011010

(c) Convert the decimal number 3.248×10^4 to a single-precision floating point binary number. **(4)**

8. (a) Use Hamilton's equations, to find the equationof motion of the simple pendulum. **(10)**

(b) Find the moment of inertia of the triangle ABC about a perpendicular to the plane through A. **(10)**

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