MIZORAM PUBLIC SERVICE COMMISSION

TECHNICAL COMPETITIVE EXAMINATIONS FOR RECRUITMENT TO THE POST OF JUNIOR GRADE OF MIZORAM PLANNING, ECONOMICS & STATISTICAL SERVICE OCTOBER, 2015

MATHEMATICS
PAPER – I

Time Allowed : 3 hours   Full Marks : 100

Figures in the margin indicate full marks for the questions.

PART – A

Attempt all questions.

1. Choose the correct answer from the following alternatives: **(10×2=20)**

(i) Consider the real function

$$f(x)=\begin{cases}1 \; \textit{if} \; \text{x is rational}\\ 0 \; \textit{if} \; \text{x is irrational}\end{cases}$$

Then

(a) f is discontinuous at every point
(b) f is continuous at every point
(c) f is discontinuous on □ except at origin
(d) None of these

(ii) The sequence $f_n(x)=x^n$

(a) Converges uniformly on [0, 1]
(b) Not convergent on [0,1]
(c) Pointwise convergent on [0, 1]
(d) None of these

(iii) Let G be a simple group of order 168. What is the number of subgroups of G of order 7?

(a) 1
(b) 7
(c) 8
(d) 28

(iv) If x and y are in Hilbert space, then

(a) $x \perp y \Rightarrow \|x+y\| = \|x-y\| > \|x\|^2 + \|y\|^2$

(b) $x \perp y \Rightarrow \|x+y\| = \|x-y\| < \|x\|^2 + \|y\|^2$

(c) $x \perp y \Rightarrow \|x+y\| = \|x-y\| = \|x\|^2 + \|y\|^2$

(d) none of these

(v) The set $M_2(□) = \left\{\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} : a,b \in □ \right\}$, under matrix multiplication and addition is

(a) A right ideal which is not a left ideal
(b) A left ideal which is not a right ideal
(c) Not an ideal of $M_2(□)$
(d) None of these

(vi) Let $T:\mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Which of following statements implies that T is bijective?

(a) Nullity (T) =n
(b) Rank (T) +Nullity (T) =n
(c) Rank (T)-Nullity (T) =n
(d) Rank (T) =Nullity (T) =n

(vii) Choose the correct one from the following

(a) Every abelian group G is not a module over the ring of integers.
(b) The kernel of the homomorphism T of an R-module has only one element
(c) Any irreducible R-module is cyclic
(d) The range of a homomorphism is a submodule.

(viii) Let $X=\{a,b,c\}$ and $B=\{\{b,c\},\{c,a\}\}$, then

(a) B is not a subset of X
(b) B cannot be a base for any topology on X
(c) B can be a base for any topology on X
(d) none of these

(ix) The space l^p is a Hilbert space if

(a) $p=1$
(b) $p=2$
(c) $p=3$
(d) $p=n$

(x) The degree of freedom for a particle moving in straight line is

(a) 0
(b) 1
(c) 2
(d) 3

PART – B

Attempt all questions.

2. Prove that the real valued function f on $\mathbb{R}^2$ defined by **(2)**

$$f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & if\ (x,y)\neq(0,0) \\ 0 & , if\ (x,y)=(0,0) \end{cases}$$

has unequal mixed partial derivatives $D_{12}f$ and $D_{21}f$ at $(0,0)$.

3. If $(R,+,\times)$ is a ring with $a^2=a\ \forall a\in R$, show that $a+a=0$ and $ab=ba\ \forall a,b\in R$. **(2)**

4. Give an example of a space which is first countable but not second countable. **(2)**

5. Prove that the angular momentum conjugate to a cyclic coordinate is conserved. **(2)**

6. Show that the sequence $\left(\dfrac{1}{n}\right)$ is a Cauchy sequence. **(2)**

7. Show that if K is compact subset of $\mathbb{R}$ and if $f: K \to \mathrm{R}$ is continuous on K, then f(K) is compact. **(2)**

8. Show that an orthonormal set is linearly independent. **(2)**

9. Consider the linear mapping $f: \mathrm{R}^2 \to \mathrm{R}^2$ defined by $f(x, y) = (3x + 4y, 2x - 5y)$.

   Find the matrix A relative to the basis {(1, 0), (0, 1)}. **(2)**

10. Show that every absolutely convergent sequence is convergent. **(2)**

11. Show that every subgroup of an abelian group is normal. **(2)**

PART – C

Attempt ***any 6 (six)*** *questions.*

12. Establish Taylor's formula for real-valued functions on a subset of $\square^n$ stating clearly the conditions to be fulfilled by a function and its domain.

    Using Taylor's formula express the function $f(x, y) = x^2 + xy + y^2$ in power of $(x-1)$ and $(y-2)$.

    (10)

13. If K is a compact subset of □ and $f: K \to \square$ is injective and continuous. Show that f^{-1} is continuous on $f(K)$. **(10)**

14. If H is a subgroup of G and N is a normal subgroup of G, then prove that

$$\frac{HN}{N} \cong \frac{H}{H \cap N}$$ **(10)**

15. Show that a Euclidian domain is always a Principal Ideal domain. **(10)**

16. Explain the principle of virtual work and discuss that d'Alembert's principle is a generalisation of the principle of virtual work for a dynamical system. **(10)**

17. Prove that the Gaussian integers $_{Z}(i) = \{a + ib \mid a, b \in Z\}$ form a Euclidean ring. **(10)**

18. Show that the sequence $< f_n >$ where $f_n(x) = \dfrac{\sin nx}{\sqrt{n}}$ is uniformly convergent on $[0, \pi]$. **(10)**

19. Prove that the product of finitely many compact spaces is compact. **(10)**

20. (a) Let G be a group of order 30. Show that a 3-Sylow subgroup or a 5-Sylow subgroup of G must be normal. **(5)**

(b) Show that the order of subgroups of a finite order group divides the order of the group. **(5)**

21. Define Poisson bracket and prove that the fundamental Poisson bracket are invariant under canonical transformation. **(10)**

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