MIZORAM PUBLIC SERVICE COMMISSION

TECHNICAL COMPETITIVE EXAMINATIONS FOR RECRUITMENT TO THE POST OF JUNIOR GRADE OF MIZORAM PLANNING, ECONOMICS & STATISTICAL SERVICE
OCTOBER, 2015

STATISTICS
PAPER – II

Time Allowed : 3 hours | Full Marks : 100

Figures in the margin indicate full marks for the questions.

PART – A
(10×2=20)

Attempt all questions.

1. A population containing N-units is divided into h strata, the i^{th} stratum containing N_i units , i = 1,2,…,h, $N_1 + N_2 + \ldots + N_h = N$. A simple random sample of n_i units is drawn from i^{th} stratum for i = 1,2,…,h. The variance of the customary estimator for the population mean is minimized for a fixed total sample size if n_i is proportional to

(a) N_i (b) S_i

(c) $N_i S_i$ (d) N_i/S_i

2. The distribution of duration of unemployment for all 18-24 year-old Americans is nearly normal with mean 12.7 weeks and standard deviation 0.3 weeks. Suppose we randomly sample 20 people from this population, ask them about the duration of their unemployment (in number of weeks), and record the sample mean. We repeat this 5,000 times, and build a distribution of sample means. What is the name of this distribution?

(a) sample distribution (b) population distribution

(c) sampling distribution (d) normal distribution

3. Let X_1, X_2, X_3 be the random observations from a populations with mean θ. Some estimators of θ are suggested below:

I. $T_1 = (X_1 - 2X_2)$

II. $T_2 = (2X_2 - X_3)$

III. $T_3 = (X_1 + X_{2+} X_3)/3$

IV. $T_4 = (X_1 + 3X_2 + X_3)/5$

Which of the above estimators are unbiased?

(a) T_1 and T_2 only (b) T_3 and T_4 only

(c) T_1, T_2, T_3, and T_4 (d) T_2, T_3 and T_4 only

4. Cluster sampling is better than the simple random sampling if the intra-class correlation coefficient is

(a) Positive and less than one
(b) Negative
(c) One
(d) Zero

5. Power of the test is given by

(a) β
(b) $2-\beta$
(c) $1-\beta$
(d) $\alpha-\beta$

6. A Statistic T_n such that $Var(T_n) \to 0$ for all θ, is consistent as an estimator of θ *as* $n \to \infty$

(a) *if and only if* $E(T_n) \to \theta$ *for all* θ

(b) *if, but not only if* $E(T_n) \to \theta$ *for all* θ

(c) *if and only if* $E(T_n) = \theta$ *for all* θ *for every* n

(d) *if and only if* $|E(T_n) - \theta| Var(T_n) \to 0$ *for all* θ

7. The value of the matrix game with the pay off matrix $A=\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ is

(a) 3
(b) 2
(c) 1
(d) 0

8. To test the equality of the population means of two normal population $N(\mu_1, \sigma_1^2)$ *and* $N(\mu_2, \sigma_2^2)$ Consider test statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where s^2 is the pooled sample variance based on samples of sizes n_1 and n_2 from the two populations. Then

(a) the distribution of t is Student's t if $\sigma_1^2 \neq \sigma_2^2$ but unknown.

(b) the distribution of t is N(0,1) if $\sigma_1^2 = \sigma_2^2 = \sigma^2$ known.

(c) the distribution of t is Student's t if $\sigma_1^2 = \sigma_2^2 = \sigma^2$ but unknown.

(d) the distribution of t is Student's t if σ_1^2 and σ_2^2 are known and distinct.

9. An estimator is the best linear unbiased estimator (BLUE) if it is the linear unbiased estimator with the

(a) largest variance
(b) smallest mean
(c) largest mean
(d) smallest variance

10. Two samples pools of votes for two candidates A and B for a public office are taken, one from among the residents of rural areas, the other from urban areas. To test whether the nature of the area is related to voting preference in this election. The most appropriate test is

(a) t – test
(b) Chi – Square test
(c) F –test
(d) Z – test

Part – B (Short answer) (5×4 = 20)

Answer the following questions:

11. Define simple random sampling and purposive sampling.

12. A random sample $(x_1, x_2, x_3, x_4, x_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ:

(i) $t_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$ (ii) $t_2 = \frac{X_1 + X_2}{2} + X_3$

(iii) $t_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$ where λ is such that t_3 is an unbiased estimator of μ.

Find λ. Are t_1 and t_2 unbiased? State giving reasons, the estimates which are best among t_1, t_2 and t_3.

13. Discuss the applicability of t-distribution.

14. Let X have a p.d.f of the form:

To test $H_0 : = 2$, against $H_1 : = 1$, use the random sample x_1, x_2 of size 2 and define a critical region $C = \{ (x_1, x_2) : 9.5 \leq x_1 + x_2 \}$.

Find (i) Power of the test

(ii) Significance level of the test.

15. What do you mean by dominance principle in the game theory?

Part – C (Descriptive) (6×10 = 60)

Answer any <u>six</u> questions:

16. Explain the following

(a) SRSWR
(b) SRSWOR
(c) Stratified sampling

17. What do you understand by point estimation? When would you say that estimate of a parameter is good? In particular, discuss the requirements of consistency and unbiasedness of an estimate. Give an example to show that a consistent estimate need not be unbiased.

18. Explain Chi-square test. If a variable X follows chi-square distribution with n degree of freedom and variable Y follows same distribution with m degree of freedom. Then, find the distribution of X+Y.

19. Define t-test and also construct the t-test for differences of two means.

20. Describe three non-parametric tests for two sample problems. Also discuss their silent features.

21. Define the following

(a) Simple and composite hypothesis

(b) Types of Errors in testing of hypothesis problem

(c) Critical region

22. What do you understand by Dominance property? Solve the following 3×5 game using dominance property.

		Player B				
		1	2	3	4	5
Player A	1	2	5	10	7	2
	2	3	3	6	6	4
	3	4	4	8	12	1

23. What do you mean by analysis of variance? Provide the complete analysis of one-way classified model.

24. Write short notes on *any two*.

(a) Pure and mixed strategy

(b) Ordinary least square estimation

(c) Heteroscedasticity

25. Prove that if $\lambda'\beta$ is estimable, its best estimate is $\lambda'\beta$, where $\hat{\beta}$ is any solution of the equations $A'A\beta = A'y$, A is the coefficient matrix and y is the observation vector.

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